Principles of Communications

ECS 332

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4.4 Switching MODEM

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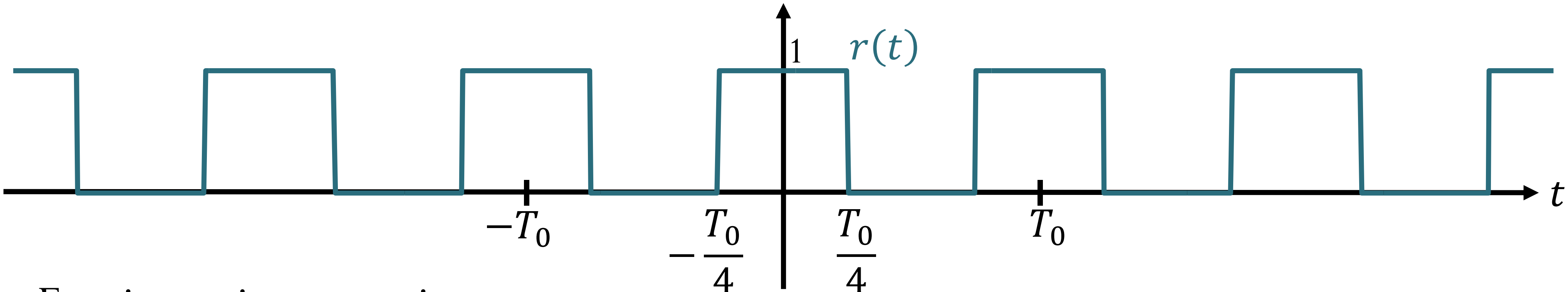
Section 4.3

- Skill 4.3.1: Know the meaning of c_0.
- Skill 4.3.2: Find the Fourier coefficients of a periodic signal when we know the Fourier transform of one period.
- Skill 4.3.3: Know the relationship between the different frequency domain representations (Fourier coefficients, line spectrum, and Fourier transform) of a periodic signal.
- **Crucial Skill 4.3.4**: Find the Fourier series expansion of the periodic train of impulses and the periodic train of rectangular pulses.
- **Crucial Skill 4.3.5**: Understand the relationship between the "switching box" and "multiplication by rectangular pulse train".

Section 4.3

- **Crucial Skill 4.3.4**: Find the Fourier series expansion of the periodic train of impulses and the periodic train of rectangular pulses.
- **Crucial Skill 4.3.5**: Understand the relationship between the "switching box" and "multiplication by rectangular pulse train".

[4.53] Square Wave



Fourier series expansion:

$$r(t) = \frac{1}{2} + \frac{1}{\pi} e^{j(2\pi f_0 t)} - \frac{1}{3\pi} e^{j(2\pi(3f_0)t)} + \frac{1}{5\pi} e^{j(2\pi(5f_0)t)} + \cdots$$
$$+ \frac{1}{\pi} e^{j(2\pi(-f_0)t)} - \frac{1}{3\pi} e^{j(2\pi(-3f_0)t)} + \frac{1}{5\pi} e^{j(2\pi(-5f_0)t)} + \cdots$$

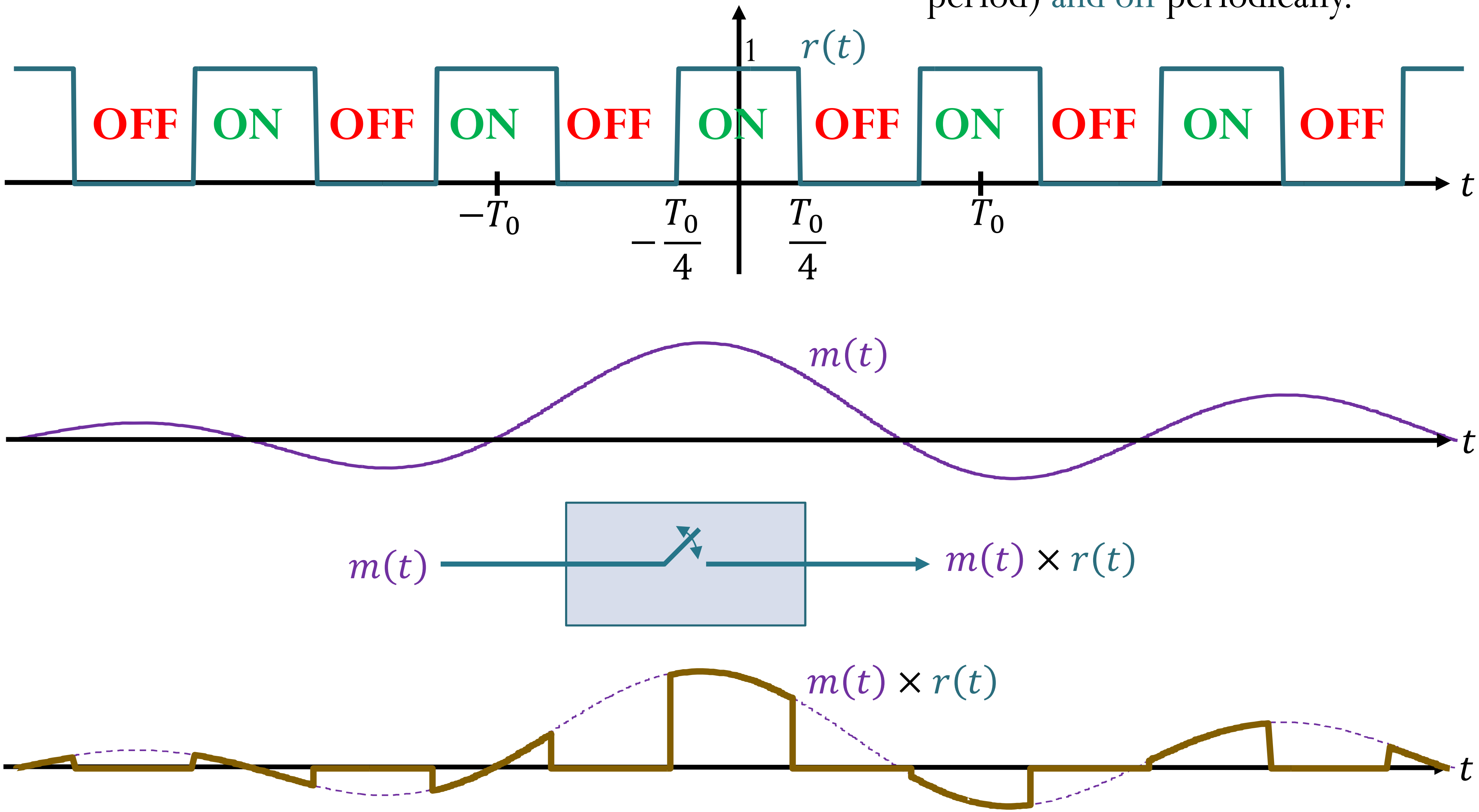
Trigonometric Fourier series expansion:

$$e^{jx} + e^{-jx} = 2\cos(x)$$

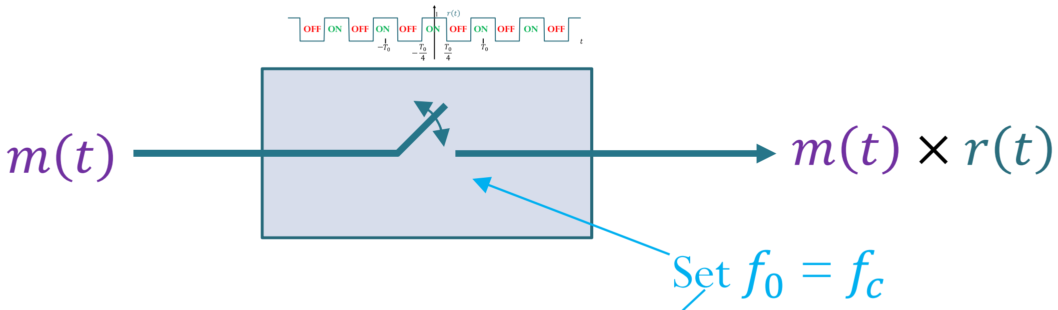
$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_0 t) - \frac{2}{3\pi}\cos(2\pi(3f_0)t) + \frac{2}{5\pi}\cos(2\pi(5f_0)t) + \cdots$$

[4.48a] Switching Operation

Multiplying a signal $m(t)$ by the square-wave $r(t)$ is equivalent to switching $m(t)$ on (for half a period) and off periodically.

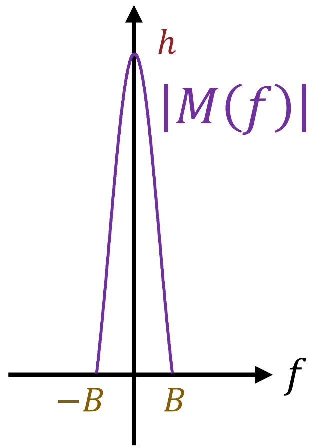


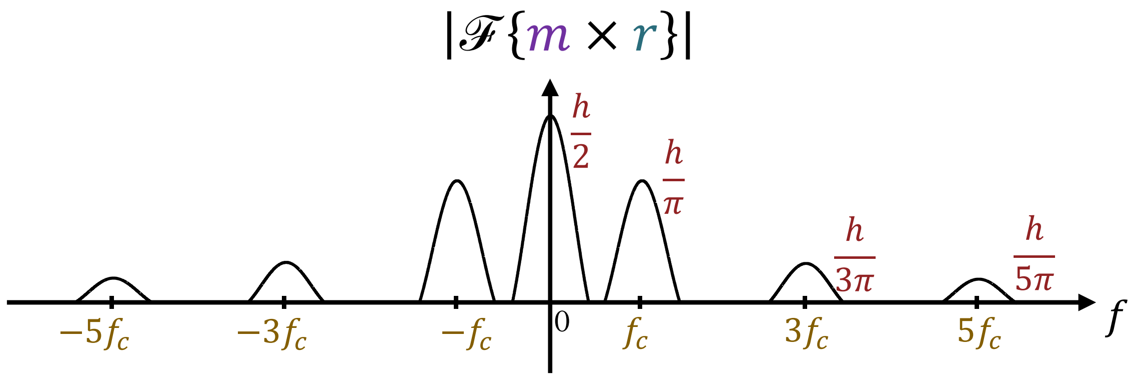
Switching Modulator



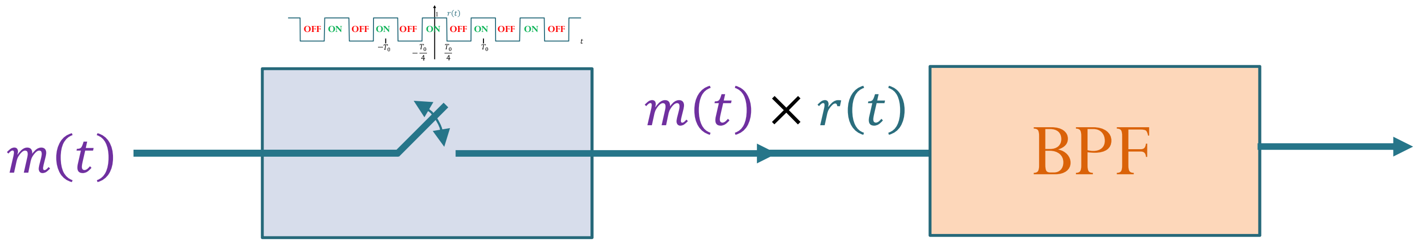
$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi(3f_c)t) + \frac{2}{5\pi}\cos(2\pi(5f_c)t) + \cdots$$

$$m(t) \times r(t) = \frac{1}{2}m(t) + \frac{2}{\pi}m(t)\cos(2\pi f_c t) - \frac{2}{3\pi}m(t)\cos(2\pi(3f_c)t) + \frac{2}{5\pi}m(t)\cos(2\pi(5f_c)t) + \cdots$$





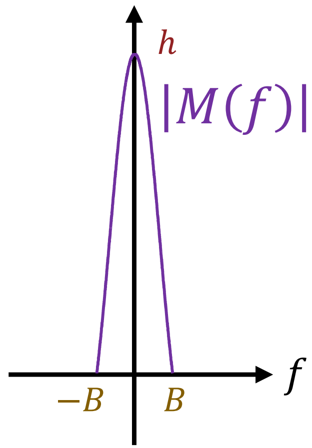
Switching Modulator

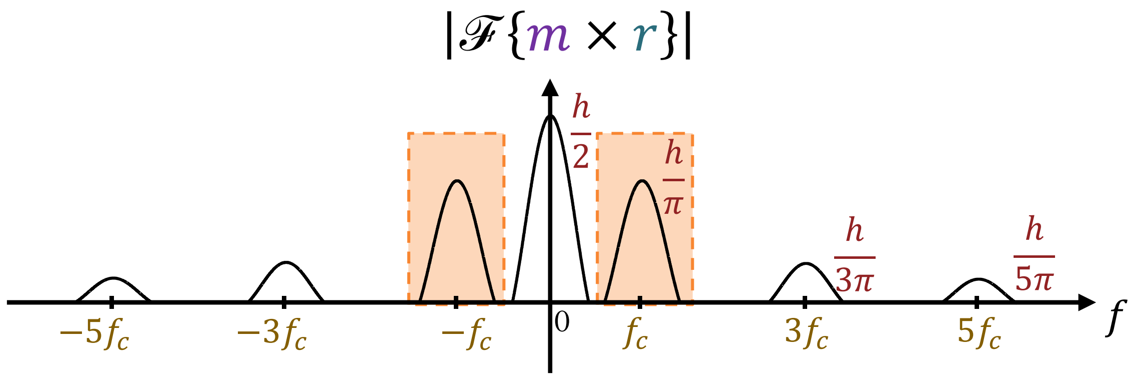


$$x(t) = \frac{2}{\pi} g m(t) \cos(2\pi f_c t)$$

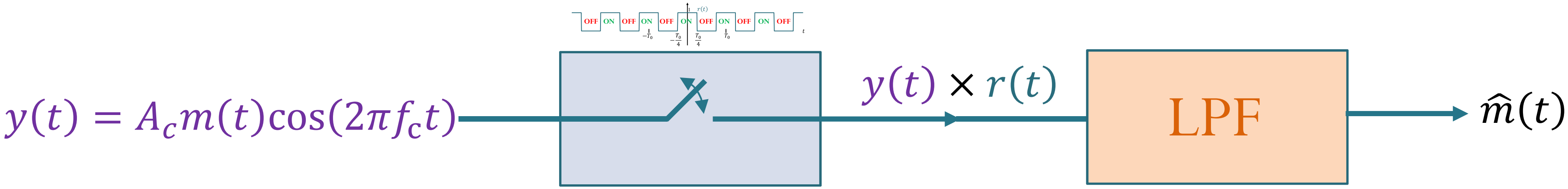
$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi(3f_c)t) + \frac{2}{5\pi}\cos(2\pi(5f_c)t) + \cdots$$

$$m(t) \times r(t) = \frac{1}{2}m(t) + \frac{2}{\pi}m(t)\cos(2\pi f_c t) - \frac{2}{3\pi}m(t)\cos(2\pi(3f_c)t) + \frac{2}{5\pi}m(t)\cos(2\pi(5f_c)t) + \cdots$$





Switching Demodulator

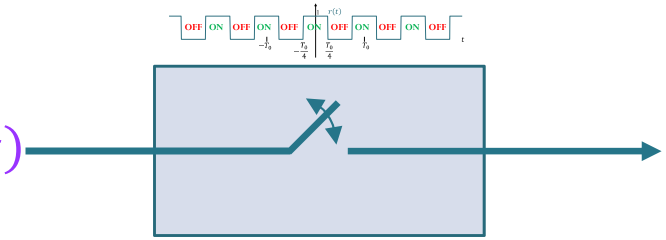


$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi(3f_c)t) + \frac{2}{5\pi}\cos(2\pi(5f_c)t) + \cdots$$

$$y(t) \times r(t) = \frac{1}{2}y(t) + \frac{2}{\pi}y(t)\cos(2\pi f_c t) - \frac{2}{3\pi}y(t)\cos(2\pi(3f_c)t) + \frac{2}{5\pi}y(t)\cos(2\pi(5f_c)t) + \cdots$$

Switching Demodulator

$y(t) = A_c m(t)\cos(2\pi f_c t)$



$y(t) \times r(t)$

$$y(t)r(t) = \frac{1}{2}y(t) + \frac{2}{\pi}y(t)\cos(2\pi f_c t) - \frac{2}{3\pi}y(t)\cos\big(2\pi(3f_c)t\big) + \frac{2}{5\pi}y(t)\cos\big(2\pi(5f_c)t\big) + \cdots$$

$$= \frac{1}{2}A_c m(t)\cos(2\pi f_c t)$$

$$+ \frac{2}{\pi}A_c m(t)\cos(2\pi f_c t)\cos(2\pi f_c t)$$

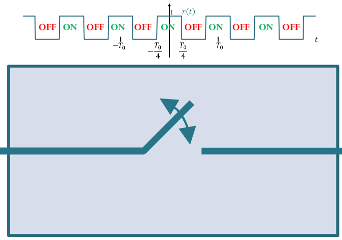
$$- \frac{2}{3\pi}A_c m(t)\cos(2\pi f_c t)\cos\big(2\pi(3f_c)t\big)$$

$$+ \frac{2}{5\pi}A_c m(t)\cos(2\pi f_c t)\cos\big(2\pi(5f_c)t\big) + \cdots$$

Now, recall that $\cos(A)\cos(B) = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$

Switching Demodulator

$y(t) = A_c m(t)\cos(2\pi f_c t)$

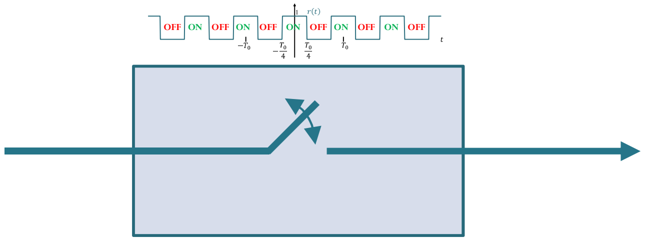


$y(t) \times r(t)$

$$\begin{aligned}
y(t)r(t) = {} & \frac{1}{2}A_c m(t)\cos(2\pi f_c t) \\
& + \frac{1}{\pi}A_c m(t)\left(1+\cos\left(2\pi(2f_c)t\right)\right) \\
& - \frac{1}{3\pi}A_c m(t)\left(\cos\left(2\pi(f_c)t\right)+\cos\left(2\pi(5f_c)t\right)\right) \\
& + \frac{1}{5\pi}A_c m(t)\left(\cos\left(2\pi(3f_c)t\right)+\cos\left(2\pi(7f_c)t\right)\right)+\cdots
\end{aligned}$$

Switching Demodulator

$y(t) = A_c m(t)\cos(2\pi f_c t)$



$y(t) \times r(t)$

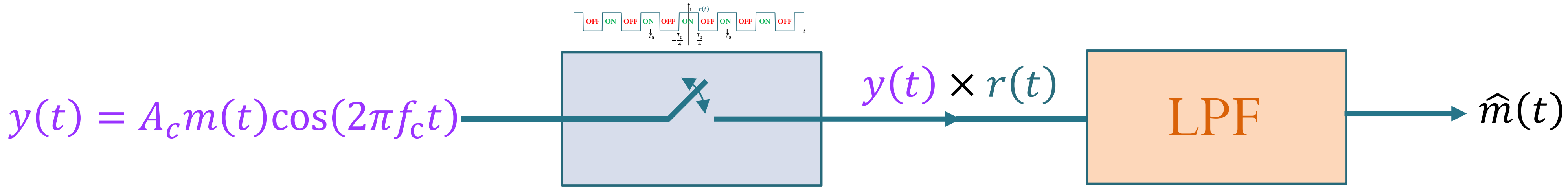
$$y(t)r(t) = \frac{1}{2}A_c m(t)\cos(2\pi f_c t)$$

$$+\frac{1}{\pi}A_c m(t) + \frac{1}{\pi}A_c m(t)\cos\left(2\pi(2f_c)t\right)$$

$$-\frac{1}{3\pi}A_c m(t)\cos\left(2\pi(2f_c)t\right) - \frac{1}{3\pi}A_c m(t)\cos\left(2\pi(4f_c)t\right)$$

$$+\frac{1}{5\pi}A_c m(t)\cos\left(2\pi(4f_c)t\right) + \frac{1}{5\pi}A_c m(t)\cos\left(2\pi(6f_c)t\right) + \cdots$$

Switching Demodulator



$$H_{\mathrm{LPF}}(f) = \begin{cases} g, & |f| < B, \\ 0, & \text{otherwise.} \end{cases}$$

$$\begin{aligned} y(t)r(t) = {} & \frac{1}{2}A_c m(t)\cos(2\pi f_c t) \\ & + \frac{1}{\pi}A_c m(t) + \frac{1}{\pi}A_c m(t)\cos(2\pi(2f_c)t) \\ & - \frac{1}{3\pi}A_c m(t)\cos(2\pi(2f_c)t) - \frac{1}{3\pi}A_c m(t)\cos(2\pi(4f_c)t) \\ & + \frac{1}{5\pi}A_c m(t)\cos(2\pi(4f_c)t) + \frac{1}{5\pi}A_c m(t)\cos(2\pi(6f_c)t) + \cdots \end{aligned}$$

$$\hat{m}(t) = \frac{A_c g}{\pi} m(t)$$

$f_c > 2B$

Section 4.4

- **Crucial Skill 4.4.1**: Able to find/draw the output of the switching box both in time and frequency domains.
- Skill 4.4.2: Able to quickly find the output of the switching modulator and switching demodulator.